MATH 3063 ABSTRACT ALGEBRA Project 4 Solutions
PROF. PAUL BAILEY April 6, 2009

Due Friday, April 3, 2009.

Copy the statement of the problem on a piece of $8\frac{1}{2} \times 11$ piece of blank computer paper, and write the solution underneath. Write neatly. Mathematics should always be written in grammatically correct English, in complete sentences.

Problem 1. Let G be a finite group of odd order. Let $f : G \to G$ be given by $f(g) = g^2$.

(a) Show that f is surjective (hint: use the Euclidean Algorithm).

(b) Show that f is bijective (hint: G is finite).

(c) Show that f is a homomorphism if and only if G is abelian.

Problem 2. Let G be a group with $K \leq H \leq G$. Show that

$$[G : K] = [G : H][H : K].$$

Problem 3. Let G be a group with $K, H \leq G$. Suppose that p, q, and r are distinct primes and that $|G| = pqr$, $|H| = pq$, and $|K| = pr$. Show that $|H \cap K| = p$.

Problem 4. Let G be a group with $H \leq G$. Show that if $[G : H] = 2$, then $H \triangleleft G$.

Problem 5. Let G be a group with $K, H \triangleleft G$. Show that $H \cap K \triangleleft G$.

Problem 6. Let G be a group with $K, H \leq G$. Set

$$HK = \{g \in G \mid g = hk \text{ for some } h \in H, k \in K\}.$$

(a) Show that if $K \triangleleft G$, then $HK \leq G$.

(b) Show that if $K \triangleleft G$ and $H \triangleleft G$, then $HK \triangleleft G$.